

# Efficient Antialiased Edit Propagation for Images and Videos

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## Abstract

Edit propagation on images/videos has become more and more popular in recent years due to simple and intuitive interaction. It propagates sparse user edits to the whole data following the policy that nearby regions with similar appearances receive similar edits. While it gives a friendly editing mode, it often produces aliasing artifacts on edge pixels. In this paper, we present a simple algorithm to resolve this artifact for edit propagation. The key in our method is a new representation called Antialias Map, in which we represent each antialiased edge pixel by a linear interpolation of neighboring pixels around the edge, and instead of considering the original edge pixels in solving edit propagation, we consider those neighboring pixels. We demonstrate that our work is effective in preserving antialiased edges for edit propagation and could be easily integrated with existing edit propagation methods such as [1, 2].

**Keywords:** Antialiasing Recovery, Edit Propagation, Antialias Map

## 1. Introduction

With the development of digital image/video cameras and online image/video sharing services (e.g. flickr, youtube), it is much easier for people to access images/videos than before. The desire to edit the appearance of image/video, such as color, brightness, tonal values, arises. One way to edit the appearance of images is to first select some regions of interest, and then apply a desired edit operation to those regions. While this is a common solution in commercial softwares such as Photoshop, selecting those regions of interest, is still a time consuming task, especially for images with complex textures. Another way is to use edit propagation methods [1, 2, 3]. In these methods, users only need to specify sparse strokes indicating specific edits (as shown in Figure 1 (a)), and those edits would be automatically propagate to the whole data following the policy that nearby regions with similar colors receive similar edits.

While edit propagation methods provide a much simpler and more convenient way for editing images/videos, it often suffers from a visible aliasing artifact. As illustrated in Figure 1, in this example, users draw a white stroke on the sky and a black one on the building, indicating an edit operation that changes color and another edit operation that keeps original color, respectively. Figure 1 (b) gives the result generated by a state-of-the-art edit propagation work [1], while it achieves the goal in most parts of the image, however, as shown in the enlarged image in (b), along the boundary of the building, we see an undesired, clear edge.

It's not surprising that edit propagation methods would produce such aliasing artifacts. This is simply because edit propagation is a per-pixel algorithm and would fail on antialiased pixels. Take Figure 1 as example, in the original image (in Figure 1

(a)), due to its antialiasing nature, the edge pixels exhibit neither the color of sky nor the color of the building, but a kind of blending between the colors of sky and the building. However, under the policy of edit propagation, those antialiased edge pixels are neither similar to the sky pixels nor to the building pixels due to color differences, this makes appearance of those edge pixels unchanged after edit propagation, leading to antialiased edges damaged, as shown in Figure 1 (b). The existence of such artifacts, has largely reduced the fidelity of results and practicality of edit propagation.

To address this issue, in this paper we introduce a novel, efficient framework to eliminate those aliasing artifacts in edit propagation. Our work is inspired by a recent work on antialiasing recovery [4], which aims at restoring antialiased edges for a range of image filters. Similar to [4], we assume that for antialiased edges in images, the value of each pixel could be seen as a linear interpolation from some nearby pixels. Based on this assumption, we introduce a novel representation, the Antialias Map, which stores the blending weights and relative positions of nearby interpolating pixels for each edge pixel. While previous works [1, 2, 3] directly consider edge pixels in solving edit propagation, we replace each edge pixel by its interpolating pixels and use those interpolating pixels in edit propagation instead. In turn, the edits of each edge pixel is obtained by an interpolation from those interpolating pixels. As shown in Figure 1 (c), our method successfully preserves the smooth edge around the boundary of the building after edit propagation. Furthermore, our method is independent of a specific edit propagation algorithm and could be integrated into any existing edit propagation methods such as [1, 2, 3]. The results demonstrate that our method effectively preserves the antialiased smooth edges without incurring large performance overhead.

The rest of the paper is organized as follows: we will first review some important related works in edit propagation and

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**Figure 1:** An example of edit propagation. (a) shows the original image and user strokes. (b) and (c) show the propagation results using the method by [1] and our method, respectively. Alias artifacts are visible in (b) along the boundary of the building. Our method successfully eliminate these artifacts, as shown in (c).

66 antialiasing recovery, respectively, in Section 2; the Antialias  
 67 Map will be introduced in Section 3; the framework and algo-  
 68 rithm details for edit propagation will be explained in Section  
 69 4; after that, results and comparisons will be given in Section 5  
 70 and conclusions will be made in Section 6.

## 71 2. Related Works

72 In this section we will review some important prior works  
 73 in edit propagation and antialiasing recovery, respectively.

### 74 2.1. Image/Video Editing

75 Image/video editing is an increasingly hot topic in comput-  
 76 er graphics in recent years. It could be generally divided into  
 77 two groups: structural editing [5, 6, 7, 8, 9, 10] and appear-  
 78 ance editing. Appearance editing includes tone editing [11, 12,  
 79 13, 14, 15, 16], colorization [17, 18, 19, 20], dehazing [21, 22,  
 80 23, 24, 25], and edge-aware editing [26, 27, 28, 29, 30, 31],  
 81 etc.. Recently, edit propagation methods [1, 2, 3, 32] allow  
 82 a simpler interaction mode for appearance editing. In these  
 83 methods, users specify edits in some sparse locations on im-  
 84 ages/videos, and those edits are automatically propagated to the  
 85 whole data satisfying the policy that nearby pixels having simi-  
 86 lar appearances receive similar edits. Usually, edit propagation  
 87 methods define affinities between pairs of pixels according to  
 88 their appearance/position distances, and different optimization  
 89 schemes are utilized to satisfy the policy. In particular, Pellaci-  
 90 ni et al. [32] approximate pixel relations using a sparse graph  
 91 and reduce edit propagation problem to solving a sparse linear  
 92 system. An and Pellacini [3] introduced a more robust algo-  
 93 rithm, which considers all-pairs affinities between pixels, and  
 94 approximates the huge affinity matrix using a low rank approx-  
 95 imation. To accelerate edit propagation, Xu et al. [1] uses a  
 96 k-d tree to organize pixels into hierarchical clusters in a high  
 97 dimensional space, instead of propagating on individual pixel-  
 98 s, they propagate on clusters which largely reduced time and  
 99 memory cost. Xiao et al. [33] employs a similar hierarchi-  
 100 cal structure for acceleration. Li et al. [2] further speeds up  
 101 edit propagation by formulating it as a function interpolation  
 102 problem. Bie et al. [34] accelerate edit propagation using stat-  
 103 ic clustering and efficient sampling scheme. Besides images  
 104 and videos, edit propagation could be also used to edit spatially  
 105 varying bidirectional reflectance distribution functions obtained  
 106 by [35, 36, 37, 38] and bidirectional texture functions [39, 40].

107 Recently, Farbman et al. [41] proposes to use diffusion distance,  
 108 instead of Euclidean distance, to define affinities between pix-  
 109 els, which better account for the global distribution of pixels.

### 110 2.2. Antialiasing Recovery

111 In computer graphics, many techniques have been proposed  
 112 to render antialiased images [42, 43], antialiased textures [44,  
 113 45] and antialiased shadows [46, 47, 48]. However, only a  
 114 few works focus on recovering smooth, antialiased edges from  
 115 aliased 2D images. Some exceptional works include Principle  
 116 Component Analysis (PCA) [49, 50] and morphological anti-  
 117 aliasing [51]. In particular, morphological antialiasing aims  
 118 at reducing aliasing artifacts for rendered images entirely us-  
 119 ing image based methods. It looks for certain patterns of dis-  
 120 continue geometry and replace them using smooth edges esti-  
 121 mated by an antialiased edge model. Image vectorization tech-  
 122 niques [52, 53, 54] convert a bitmap image to a vectorized im-  
 123 age, which could also be used to antialias certain types of im-  
 124 ages. Recently, Yang et al. [4] introduced a method for recover-  
 125 ing antialiased edges destroyed by a range of non-linear image  
 126 filters. In this work, an analytic edge model is estimated using  
 127 the original image, and is applied to the filtered image to re-  
 128 move aliasing artifacts. It works well for many non-linear im-  
 129 age filters such as intensity thresholding, tone mapping, color  
 130 to gray and so on, however, since it requires perfect pixel cor-  
 131 respondence between the original and filtered images, it cannot  
 132 handle filters like Gaussian blurring. Besides, it's not clear how  
 133 to extend this method to edit propagation.

134 Compared to the conference paper [55], We have extended  
 135 our framework to handle interpolation based edit propagation.  
 136 This is a significant new contribution compared to [55], since  
 137 we have demonstrated the proposed Antialias Map is not limit-  
 138 ed to optimization based edit propagation, however, it could al-  
 139 so be used for interpolation based edit propagation. This demon-  
 140 strates that the proposed Antialias Map is independent with spe-  
 141 cific edit propagation methods and could be potentially com-  
 142 bined with any edit propagation methods.

## 143 3. Antialias Map

144 As mentioned before, since antialiased edges in images are  
 145 often smooth, we assume the value of an edge pixel could be  
 146 approximated by a linear interpolation of some nearby pixels.  
 147 We present Antialias Map to store those edge pixels. Besides,

in Antialias Map, for each edge pixel ,we also store the information of its neighboring interpolating pixels, including both interpolating weights and relative positions. For videos, we stored an Antialias Map for every frame. Since our work is built upon the antialiasing recovery work of [4], to make our paper self-contained, before introducing the details of Antialias Map, we will first explain some necessary backgrounds in [4] in Section 3.1.

### 3.1. Antialiasing Recovery

Images often have smooth, antialiased edges. However, these desired properties will be destroyed by a range of nonlinear image filters, such as intensity thresholding, tone mapping, etc.. After applying those image filters, smooth boundaries become zigzag like. Yang et al. [4] proposed a technique to remove these aliasing artifacts in filtered images. Their method proceeds in several steps:

**Edge model.** For each pixel  $i$  in the original image, they choose the two extremum colors  $c_j$  and  $c_k$  ( $j,k$  are corresponding pixels) in the principle direction of color space from the neighboring 8 pixels (in  $3 \times 3$  size neighborhood). The principle direction is determined using an Expectation Maximization (EM) scheme. Using extremum colors to reconstruct the color  $c_i$  of pixel  $i$ , the interpolation weights  $\alpha_{ij}, \alpha_{ik}$  could be determined by minimizing:

$$d_i = \|(\alpha_{ij}c_j + \alpha_{ik}c_k) - c_i\| \quad (1)$$

where it satisfies  $\alpha_{ij} + \alpha_{ik} = 1$ .

**Probability of lying on edges.** After that, they estimate the probability of each pixel that it lies on an edge. For each pixel  $i$ , They define an edge strength  $e_i$ , which is the product of the Sobel edge detector at both the original image and the filtered image. The probability value of a pixel lying on an edge is defined as:

$$\beta_i = G(d_i, \sigma_d)(1 - G(e_i, \sigma_e)) \quad (2)$$

where  $G(d, \sigma)$  is a 1D Gaussian defined as  $\exp(-d^2/\sigma^2)$ ,  $d_i$  is the residual distance defined in Equation 1,  $\sigma_d$  and  $\sigma_e$  are two controllable parameters.  $\beta_i$  is set as zero if  $e_i > 3\sigma_e$ .

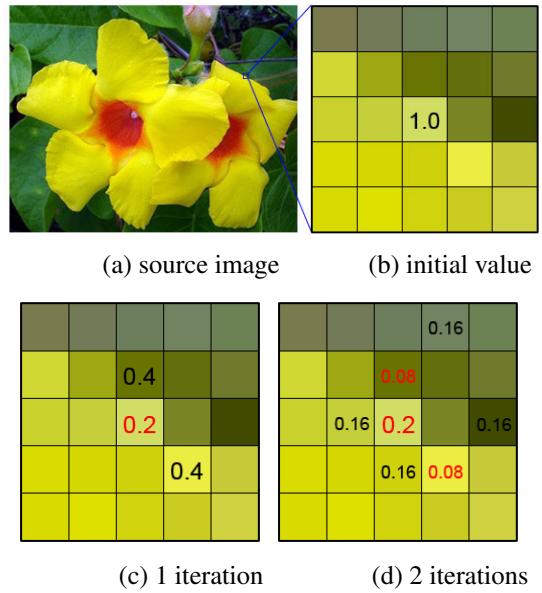
**Recovery the filtered image.** Denote  $f_i$  is the color value of pixel  $i$  on the filtered image. The recovered color value  $r_i$  could be obtained by solving the linear system below:

$$r_i = \beta_i(\alpha_{ij}r_j + \alpha_{ik}r_k) + (1 - \beta_i)f_i \quad (3)$$

This is a large sparse linear system and could be solved efficiently by a few iterations using the Jacobi method.

### 3.2. Compute Antialias Map

As discussed in Section 3.1, in [4], the value of each antialiased edge pixel is approximated by a blending of 2 neighbors by pixels in the  $3 \times 3$  neighborhood. Results are progressively refined by iterations of Equation 3. Instead of using a  $3 \times 3$  neighborhood, Antialias Map approximates the value of each pixel by a blending of pixels from a larger neighborhood:



**Figure 2:** Antialias Map construction. (a) is the source image; (b),(c) and (d) give the Antialias Map of a certain pixel after 0,1,2 iterations, respectively. Divisible pixels are colored black, while indivisible pixels are colored red.

$$c_i \approx \sum_j w_{ij}c_j \quad (4)$$

where  $j$  is the *interpolating pixel* in the neighborhood of  $i$ , and  $w_{ij}$  is the *interpolating weight* from pixel  $i$  to  $j$ , and satisfies  $\sum_j w_{ij} = 1$ . Note that  $w_{ij}$  does not necessarily equal to  $w_{ji}$ . Also note that Equation 4 is not an optimization target, and the interpolating weights are not solved from Equation 4. Instead, the interpolating weights are computed through an iterative approach, which will be explained in detail below.

Antialias Map has two advantages over the edge model proposed in [4]. First, since it uses a larger neighborhood to approximate an antialiased pixel, it leads to a more accurate approximation; Secondly, the Antialias Map only depends on the structure of original image itself, it could be computed and stored before edit propagation, so it avoids the cost of iterations at run-time edit propagation stage. Antialias Map stores all interpolating weights  $w_{ij}$ , and it is sparse since it only considers those edge pixels (e.g. whose edge strength  $\beta_i$  is non-zero) and it only stores non-zero weights. Specifically, we store a set of triples  $(\Delta x_{ij}, \Delta y_{ij}, w_{ij})$  for each edge pixel  $i$ . Here  $j$  is its interpolating pixel,  $\Delta x_{ij}, \Delta y_{ij}$  and  $w_{ij}$  are the x,y position offset and interpolating weight from  $i$  to  $j$ , respectively. In the following parts, we will explain how to compute the Antialias Map in detail.

**Initialization.** In this step, we first use [4] to obtain the two extremum neighbors  $j, k$ , the blending factors  $\alpha_{ij}, \alpha_{ik}$  and the edge probability  $\beta_i$  for each pixel  $i$ . We have already explained how to compute those values in Section 3.1. Care must be taken when computing the edge probability  $\beta_i$ . In [4], it defines edge strength of each pixel as the product of Sobel edge detector on

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**Initialization**

For all pixels  $i$   
 Compute the blending factors  $\alpha_{ij}, \alpha_{ik}$ ,  
 and the edge probability  $\beta_i$ .

End For

---

**Computation**

Step 1: Antialias Map  $S_i = \{\{0, 0, 1\}\}$

Step 2:

for each triple  $\{\Delta x_{ij}, \Delta y_{ij}, w_{ij}\}$  in  $S_i$   
 if the pixel  $j$  is divisible and  $\beta_j w_{ij} > \sigma_a$   
 fetch blending factors  $\alpha_{jk_1}, \alpha_{jk_2}$  and edge probability  $\beta_j$ ;  
 update the weight of pixel  $j$  to  $(1 - \beta_j)w_{ij}$ ;  
 mark pixel  $j$  as indivisible;  
 add pixel  $k_1$  and  $k_2$  to Antialias Map  $S_i$ , with weights  
 $w_{ik_1} = \alpha_{jk_1} \beta_j w_{ij}, w_{ik_2} = \alpha_{jk_2} \beta_j w_{ij}$ ,  
 mark these two pixels as divisible.

end if

end for

if iteration number reaches  $N$   
 End.  
 else  
 go back to Step 2.  
 end if

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**Table 1:** Pseudocode for Antialias Map Construction.

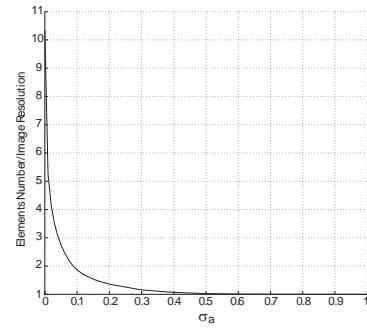
both original and filtered images, which means it requires to obtain the aliased filtered image before antialiasing recovery. We observe that in edit propagation, the appearances are changed smoothly, so that the propagated result images have roughly the same structure as the original images. To avoid the cost to generate an aliased edit propagation result, we make a modification, instead, we define the edge strength as the Sobel edge detector only on the original image. Once the edge strength is computed, we use Equation 2 to compute edge probability  $\beta_i$ . Note that only the pixels with non-zero  $\beta_i$  are considered as antialiased edge pixels and stored in Antialias Map. The pixels with zero value of  $\beta_i$  are considered as non-edge pixels.

**Constructions.** Similar to [4], we construct Antialias Map by a few iterations. However, they obtain the final antialiased results through iterations, but we obtain Antialias Map through iterations, which could be precomputed and stored before edit propagation. For each antialiased edge pixel  $i$ , the Antialias Map starts with a set containing only one triple:

$$S_i = \{\{0, 0, 1\}\} \quad (5)$$

This means that the value of the pixel  $i$  could be seen as the value of itself multiplied by weight 1.0, which is definitely true. We also illustrate this iteration process in Figure 2. As shown in Figure 2 (b), now the Antialias Map only contains itself with weight 1.0. And this pixel is marked as divisible, which is painted using black color in Figure 2.

At each iteration, we expand each divisible pixel (e.g.  $j$ ) into 3 pixels. These 3 pixels are the two neighboring extremum pixels (e.g.,  $k_1$  and  $k_2$ ) and itself (e.g.  $j$ ), whose corresponding weights are defined in Equation 3. Specially, the weight of  $j$  is replaced by  $(1 - \beta_j)w_{ij}$  and  $j$  is marked as indivisible; the two newly added extremum pixels are marked as divisible, and their weights are set as  $w_{ik_1} = \alpha_{jk_1} \beta_j w_{ij}$  and  $w_{ik_2} = \alpha_{jk_2} \beta_j w_{ij}$ , respectively. At the next iterations, we recursively find the di-



**Figure 3:** the size of Antialias Map to the size of image as a function of threshold  $\sigma_a$ . This curve is generated from a 240K photographed image (the image in Figure 1) and using maximum iteration number  $N = 4$ .

visible pixels and expand them to new pixels.

Let's also take Figure 2 as example and explain this process in detail. For simplicity, we assume that for all the pixels, the edge probability  $\beta$  is 0.8 and the blending factor  $\alpha$  is 0.5. After the first iteration, the center pixel is expanded to 3 pixels, so that the Antialias Map grows to contain 3 triples (as shown in Figure 2 (c) ):

$$S_i = \{\{0, 0, 0.2\}, \{0, -1, 0.4\}, \{1, 1, 0.4\}\} \quad (6)$$

After the second iterations, similarly, the newly added 2 pixels in first iteration are both expanded to 3 pixels, so that the Antialias Map grows to contain 7 triples (as shown in Figure 2 (d) ):

$$S_i = \{\{0, 0, 0.2\}, \{0, -1, 0.08\}, \{1, 1, 0.08\}, \{-1, 0, 0.16\}, \{1, -2, 0.16\}, \{0, 1, 0.16\}, \{2, 0, 0.16\}\} \quad (7)$$

Notice that at all iterations, the sum of weights equals to one. From an algebraic aspect, Antialias Map could also be treated as expanding Equation 3 to multiple variables. The triples in Antialias Map will extend to  $(2N + 1) \times (2N + 1)$  neighborhood after  $n$  iterations.

**Stop Criterion.** The size of the Antialias Map grows as we iterate. We define 2 criterions to stop the recursive iteration:

- When iteration number reaches a predefined number  $N$ ;
- When the result product(product of the interpolation weight of a divisible pixel  $w_{ij}$  and its edge probability  $\beta_j$ ) is smaller than a predefined threshold  $\sigma_a$ .

The pseudocode of Antialias Map construction is given in Table 1. We have also tested how two parameters influence the performance of our algorithm. Figure 3 illustrates the size of Antialias Map to the size of image as a function of weight threshold  $\sigma_a$ . Setting  $\sigma_a = 0$  means the iteration stops only when it reaches the largest iteration number  $N$ , while setting  $\sigma_a = 1$  means no iteration. As shown in Figure 3, when increasing  $\sigma_a$  from 0 to 1, the size of Antialias Map decreases rapidly.

## 285 4. Improved Framework of Edit Propagation

286 In this section we will discuss how to use Antialias Map in  
 287 the pipeline of edit propagation to remove the aliasing artifacts.  
 288 In edit propagation, users specify edits in some sparse locations  
 289 on images/videos, and those edits are automatically propagated  
 290 to the whole data satisfying the policy that nearby pixels having  
 291 similar appearances receive similar edits. Usually, they define  
 292 a feature vector for each pixel, usually a 5-dimensional vector  
 293 , which combines color (e.g. 3D), pixel position (e.g. 2D). For  
 294 videos, another dimensional is added to account for time. The  
 295 affinity between every two pixels are defined by the Euclidean  
 296 distance between their feature vectors, which is then used to  
 297 guide the propagation. Commonly, edit propagation methods  
 298 could be divided into two groups, depending on which scheme  
 299 is used to formulate the problem: optimization based [1, 3] and  
 300 interpolation based [2]. We show that Antialias Map could be  
 301 used in both groups for antialias recovery.

### 302 4.1. Optimization based Edit Propagation

303 **Backgrounds.** As mentioned above, the affinity value between  
 304 two pixels  $i, j$  is usually defined as:

$$z_{ij} = \exp\left(-(\mathbf{f}_i - \mathbf{f}_j)^2\right) \quad (8)$$

305 where  $\mathbf{f}_i$  is the feature vector of pixel  $i$ , which is defined as a 5D  
 306 vector for images:

$$\mathbf{f}_i = (c_i/\sigma_c, p_i/\sigma_p) \quad (9)$$

307 where  $c_i, p_i$  is the color in LAB color space and the pixel po-  
 308 sition of pixel  $i$ , respectively.  $\sigma_c$  and  $\sigma_p$  are two parameters to  
 309 control the relative propagating distance.

310 In [3], edit propagation is formulated as an optimization  
 311 problem. Solving propagated edits  $e$  is deduced to minimize  
 312 the energy function below:

$$\sum_{i,j} b_j z_{ij} (e_i - g_j)^2 + \lambda \sum_{i,j} z_{ij} (e_i - e_j)^2 \quad (10)$$

313 where  $i, j$  enumerates all pixels;  $b_j$  is 1 when pixel  $j$  is covered  
 314 by stroke and is 0 elsewhere;  $g_j$  is the user specified edit at pix-  
 315 el  $j$ ;  $e_i$  is the propagated edit at pixel  $i$  that we want to solve.  
 316 The first term accounts for how it satisfies user input while the  
 317 second term accounts for the edit propagation policy that simi-  
 318 lar pixels receive similar edits.  $\lambda$  is used to control the relative  
 319 weight between the two terms and is usually set to  $\sum_j b_j / \sum_j 1$   
 320 to make the two terms have roughly the same contributions.

321 Since the energy function in Equation 10 is quadratic, min-  
 322 imizing it is equivalent to solving a linear system defined by  
 323 a large affinity matrix. Therefore, they used low rank colum-  
 324 n sampling to approximate the affinity matrix and further pro-  
 325 posed an approximated algorithm to fast find a solution. To  
 326 accelerate edit propagation and extend it to videos, Xu et al. [1]  
 327 proposed to use k-d tree to organize pixels into hierarchical  
 328 clusters. Instead of propagating on pixels, they propagate on  
 329 clusters, whose number is much smaller than the number of  
 330 pixels, thus acceleration is achieved. Finally, edits of individual

331 pixels are obtained by multi-linear interpolation from clusters.  
 332 They also adopted an optimization based method to solve for  
 333 edit propagation.

334 **Modified Formulation.** As illustrated in the teaser image, tra-  
 335 ditional edit propagation produces artifacts on object bound-  
 336 aries. This artifact could be easily explained. Assume a very  
 337 simple image composed of 2 region, one red region and another  
 338 blue region. The edge pixels along the boundary of the two  
 339 regions would appear yellow due to antialiasing. Suppose user  
 340 specifies some edits on the red region, it is also desired to prop-  
 341 agate the edits to the edge pixels with some weight according to  
 342 antialiasing opacity. However, since the edge pixels appearance  
 343 yellow, it exhibits a large difference to pixels in the red region,  
 344 hence would not receive any propagated edits.

To address this issue, we use Antialias Map, in which, the yellow edge pixels would be represented by a linear blending of some red and blue neighboring pixels. Instead of propagating to the edge pixels, we propagate to the neighboring interpolating pixels, and obtain the edit of edge pixel by blending the edits from the interpolating pixels. Mathematically, we modify the formulation in Equation 10 to:

$$\sum_{i,j} b_j \gamma_j z_{ij} (e'_i - g'_j)^2 + \lambda \sum_{i,j} \gamma_j z_{ij} (e'_i - e'_j)^2 \quad (11)$$

345 where  $i, j$  enumerates all interpolating pixels;  $\gamma$  considers the  
 346 multiplicity of pixel  $i$  serving as interpolating pixels, which is  
 347 defined as  $\gamma_i = \sum_k w_{ki}$ ;  $g'_j$  is defined as  $g'_j = \sum_k w_{kj} g_j / \sum_k w_{kj}$ .

348 The modified energy function has the same form as the orig-  
 349 inal energy function in Equation 10, so that it could be solved  
 350 in the same way using either low rank column sampling [3] or  
 351 k-d tree clustering [1].

352 **Interpolation.** After solving for the edits  $e'$  on the interpolating  
 353 pixels in Equation 11, it is easy to obtain the edits on the edge  
 354 pixels through interpolation:

$$e_i = \sum_j w_{ij} e'_j \quad (12)$$

### 355 4.2. Interpolation based Edit Propagation

356 **Backgrounds.** While most works adopt an optimization based  
 357 method to solve edit propagation, Li et al. [2] proposed a dif-  
 358 ferent approach. They observe that the edits span in the high  
 359 dimensional feature space form a smooth function, which could  
 360 be approximated well by function interpolations. Therefore,  
 361 they use sum of RBFs (Radial Basis Function) to approximate  
 362 edits:

$$e_i \approx \sum_m a_m G(\|\mathbf{f}_i - \mathbf{f}_m\|) \quad (13)$$

363 where  $m$  iterates over all RBFs;  $G$  is RBF Gaussian function;  
 364  $a_m, \mathbf{f}_m$  is the  $m$ -th RBF coefficient and center, respectively. The  
 365 centers of RBFs are randomly selected from the pixels covered  
 366 by user stroke. The coefficients of RBFs are solved by mini-  
 367 mizing the sum of differences on user specified edits:

$$\sum_j (g_j - \sum_m a_m G(\|\mathbf{f}_j - \mathbf{f}_m\|))^2 \quad (14)$$

368 where  $j$  iterates over all pixels covered by user strokes. To re-  
 369 strict the coefficients to be non-negative, they use a non-negative  
 370 least square solver.

371 **Modified Formulation.** The above formulation would also  
 372 produce aliasing artifacts on object boundaries. To remove the  
 373 artifacts using Antialias Map, similarly, we build the smooth  
 374 function over the interpolating pixels, instead of the original  
 375 pixels. Equation 14 is modified to:

$$\sum_j \gamma_j (g'_j - \sum_m a_m G(\|\mathbf{f}_j - \mathbf{f}_m\|))^2 \quad (15)$$

376 where  $j$  iterates over all interpolating pixels that have contri-  
 377 butions to user stroke pixels;  $\gamma_j$  considers the multiplicity of  
 378 pixel  $j$  serving as interpolating pixels, which is defined as  $\gamma_j =$   
 379  $\sum_k w_{kj}$ ;  $g'_j$  is defined as  $g'_j = \sum_k w_{kj} g_j / \sum_k w_{kj}$ , where  $k$  is iter-  
 380 ating over user stroke pixels.

381 After solving for the RBF coefficients, we use Equation 13  
 382 to obtain the edits on interpolating pixels. Lastly, we use Equa-  
 383 tion 12 to obtain the edits on the edge pixels.

## 384 5. Comparisons and Results

### 385 5.1. Comparisons

386 **Comparison of weight threshold  $\sigma_a$ .** In Figure 4, we have  
 387 compared edit propagation results generated by Xu et al. [1]  
 388 and by our method with different weight threshold  $\sigma_a$ . From  
 389 the results, we can see artifacts using the method by Xu et al.,  
 390 where the pixels along the boundary of the toy undesirably ap-  
 391 pear green. Using a large value of  $\sigma_a$  (e.g.  $\sigma_a = 0.8, 0.4$ ) still  
 392 produce these artifacts. But using a relatively small value of  $\sigma_a$   
 393 (e.g.  $\sigma_a = 0.1, 0.0$ ) fully removes the artifacts.

394 **Comparison of maximum iteration number  $N$ .** In Figure 5,  
 395 we have compared edit propagation results generated by Xu  
 396 et al. [1] and by our method with different maximum iteration  
 397 number  $N$ . From the results, we can see that using a relatively  
 398 large value of  $N$  (e.g.  $N = 4, 8$ ) could produce smooth transi-  
 399 tions along boundaries.

### 400 5.2. Results

401 All these results and performance are obtained using a con-  
 402 sumer level PC with a 3.0GHz Intel Core2Duo CPU and 4GB  
 403 RAM. As demonstrated in the comparisons, setting  $\sigma_a = 0.1$   
 404 and  $N = 4$  already leads to very good results. So in our imple-  
 405 mentation, we fix  $\sigma_a = 0.1$  and  $N = 4$ . These two parameters  
 406 could still be adjusted for better performance or accuracy. In  
 407 our experiment, for a single image, the total size of Antialias  
 408 Map (e.g. the total number of triples) is usually about  $1.5 - 2.0$   
 409 times of the image resolution. So that it only needs small extra  
 410 space to store the Antialias Map.

411 In Figure 6, we give 2 image results generated by the k-d  
 412 tree approach [1] and by our method. In Figure 7, we give 2  
 413 image results generated by the RBF interpolation approach [2]  
 414 and by our method. In Figure 8, we give 1 image result gener-  
 415 ated by AppProp [3] and by our method. In Figure 9, we com-  
 416 pare a video example using the k-d tree approach [2] and using  
 417 our method, respectively. In all these examples, after applying

418 our methods, the aliasing artifacts along the object boundaries  
 419 are successfully removed. The performance value is reported in  
 420 Table 2. Note that the time cost reported for the video example  
 421 in Table 2 is the time for processing the whole video (all the  
 422 frames). It could be substantially accelerated for fast preview-  
 423 ing purposes, when users desire to see a single (or a few) frames  
 424 of the video, and only the pixels on the previewing frames need  
 425 to be propagated.

## 426 6. Conclusion

427 In this paper we have presented a novel, efficient approach  
 428 to remove aliasing artifacts in edit propagation. we introduced  
 429 a novel representation, the Antialias Map, to store the blending  
 430 weights and relative positions of nearby interpolating pixels for  
 431 each edge pixel. While previous works [1, 2, 3] directly consid-  
 432 er edge pixels in edit propagation process, instead, we replace  
 433 each edge pixel by its interpolating pixels and consider those  
 434 interpolating pixels in edit propagation process. Our method is  
 435 independent of a specific edit propagation algorithm and could  
 436 be integrated into any existing edit propagation methods such as  
 437 [1, 2, 3]. The results demonstrates that our method effectively  
 438 and efficiently restores the antialiased smooth edges.

439 There are some works that we would like to address in the  
 440 future. First, we currently deal with videos frame by frame, and  
 441 for each frame we use a 2D Antialias Map. We would like to  
 442 explore methods to extend Antialias Map to a 3D representa-  
 443 tion so that it could also handle motion blurs in the temporal di-  
 444 mension; Secondly, we would like to investigate how Antialias  
 445 Map could be used for other image related applications, such as  
 446 image compositing [56, 57, 58, 59] and non-photorealistic ren-  
 447 dering [60], since it is also desired to preserve antialiased edges  
 448 when compositing new images.

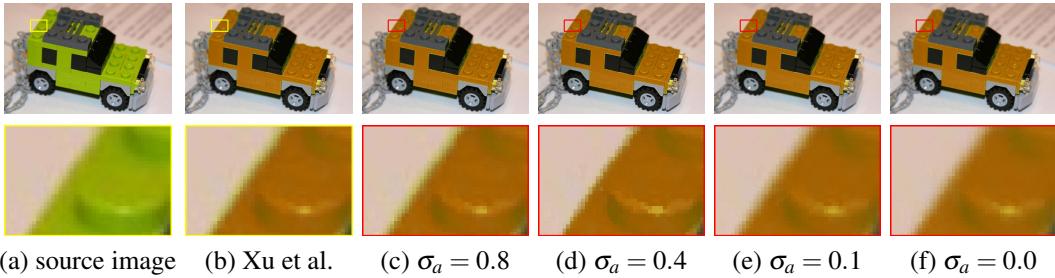
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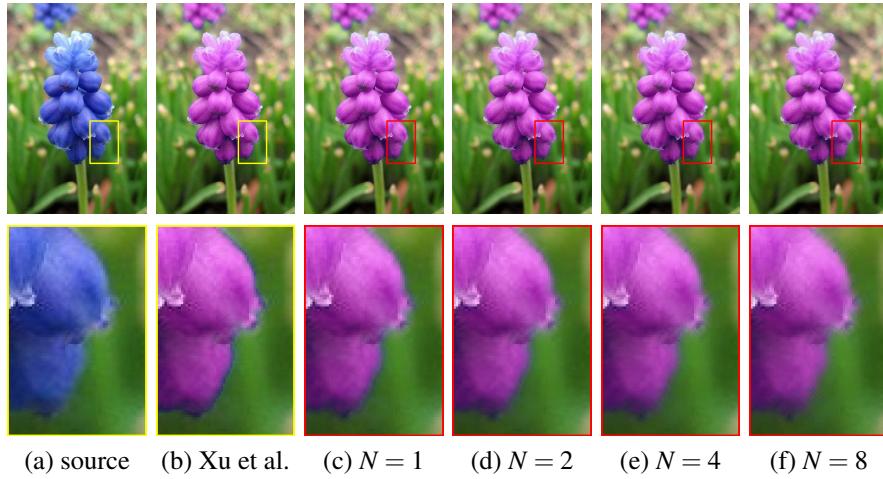
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(a) source image (b) Xu et al. (c)  $\sigma_a = 0.8$  (d)  $\sigma_a = 0.4$  (e)  $\sigma_a = 0.1$  (f)  $\sigma_a = 0.0$

**Figure 4:** Comparison of edit propagation results generated by Xu et al. [1] and by our method with different weight threshold  $\sigma_a$ . (a) is the source image  $O$ . (b) is the edit propagation result of [1], artifacts can be found along the boundaries. (c)–(f) are results using our algorithm with  $\sigma_a = 0.8, 0.4, 0.1, 0.0$ .



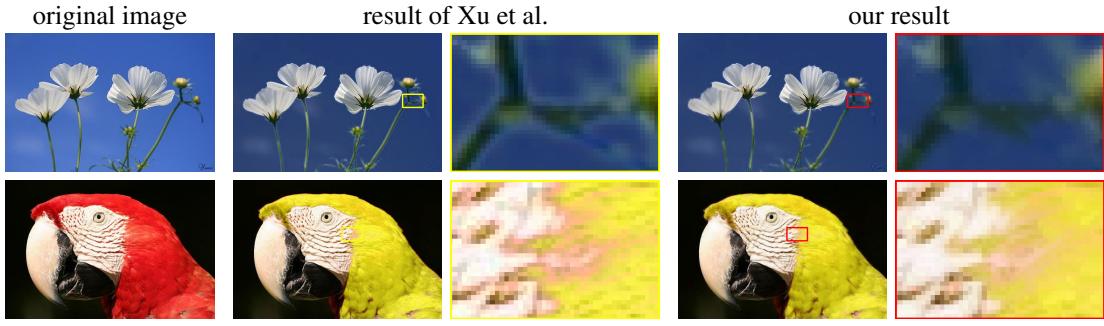
(a) source (b) Xu et al. (c)  $N = 1$  (d)  $N = 2$  (e)  $N = 4$  (f)  $N = 8$

**Figure 5:** Comparison of edit propagation results generated by Xu et al. [1] and by our method with different maximum iteration number  $N$ . (a) is the source image. (b) is the edit propagation result generated by Xu et al.. Notice that artifacts can be found along the boundaries. (c)–(f) are results using our method with  $N = 1, 2, 4, 8$ , respectively.

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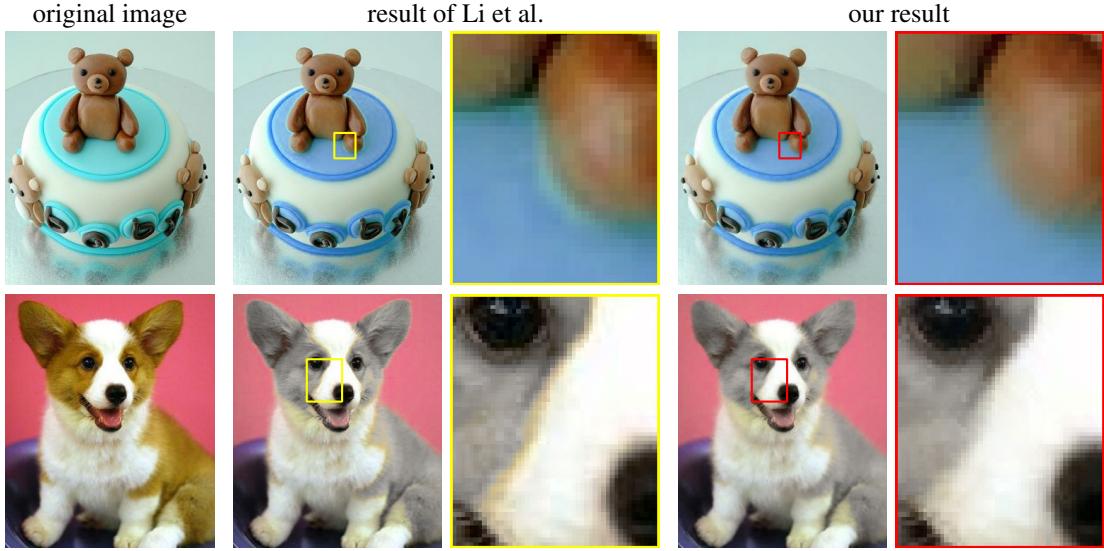
| Data              | toy<br>(Fig. 4) | flower<br>(Fig. 5) | cake<br>(Fig. 7) | dog<br>(Fig. 7) | branch<br>(Fig. 6) | parrot<br>(Fig. 6) | sky<br>(Fig. 1) | bird<br>(Fig. 8) |
|-------------------|-----------------|--------------------|------------------|-----------------|--------------------|--------------------|-----------------|------------------|
| Type              | image           | image              | image            | image           | image              | image              | image           | video            |
| Resolution        | 120K            | 120K               | 120K             | 120K            | 150K               | 150K               | 240K            | 30M              |
| Frame Num         | -               | -                  | -                | -               | -                  | -                  | -               | 400              |
| K-d tree          | time<br>memory  | 22ms<br>8MB        | 23ms<br>8MB      | 17ms<br>8MB     | 25ms<br>8MB        | 28ms<br>8MB        | 24ms<br>8MB     | 8s<br>22MB       |
| Improved k-d tree | time<br>memory  | 40ms<br>9MB        | 42ms<br>9MB      | 32ms<br>9MB     | 45ms<br>9MB        | 45ms<br>9MB        | 47ms<br>9MB     | 13s<br>24MB      |
| RBF               | time<br>memory  | 16ms<br>1MB        | 17ms<br>1MB      | 13ms<br>1MB     | 20ms<br>1MB        | 21ms<br>1MB        | 19ms<br>1MB     | 4s<br>1MB        |
| Improved RBF      | time<br>memory  | 32ms<br>1MB        | 30ms<br>1MB      | 25ms<br>1MB     | 38ms<br>1MB        | 32ms<br>1MB        | 36ms<br>1MB     | 8s<br>1MB        |

**Table 2:** Performance comparison between the *k-d* tree method [1], our method combined with the *k-d* tree approach, RBF method [2] and our method combined with the RBF method. Both running time and memory cost are reported.

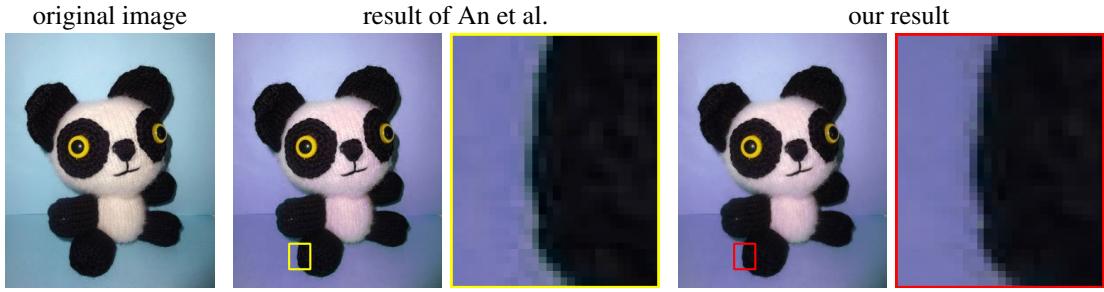


**Figure 6:** Results generated by Xu et al. [1] and by our method. The first column give the original images; the second and third columns are results generated by Xu et al.; the fourth and fifth columns are results generated by our method.

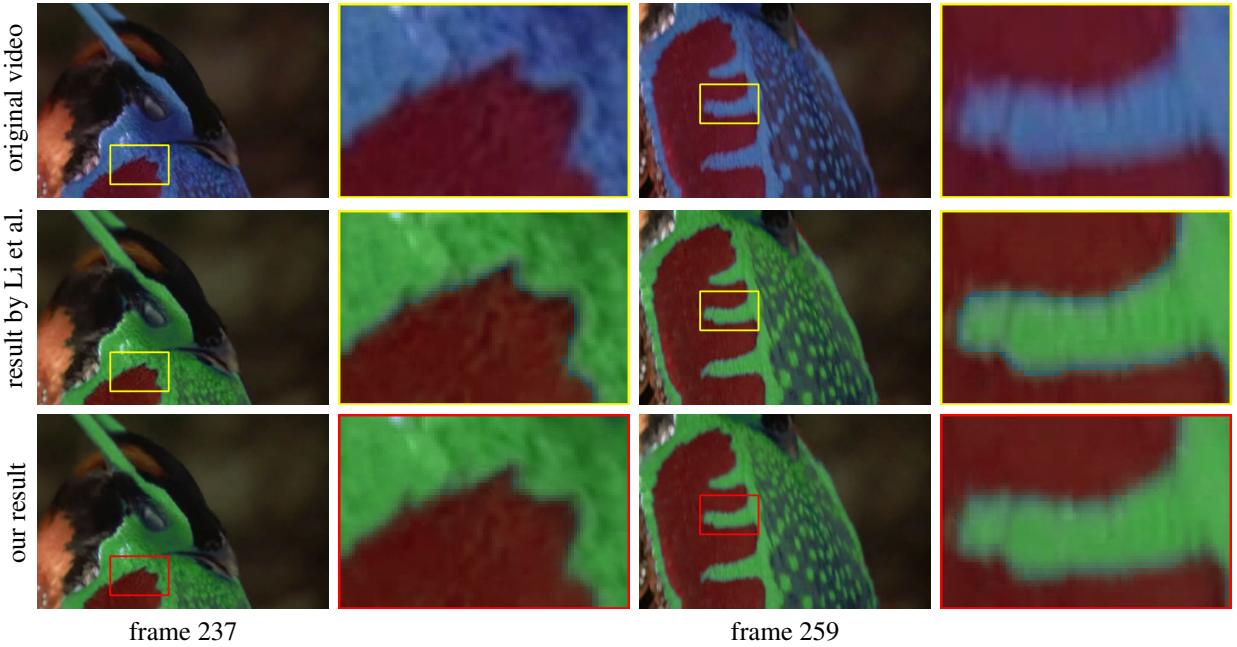
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**Figure 7:** Results generated by Li et al. [2] and by our method. The first column give the original images; the second and third columns are results generated by Li et al.; the fourth and fifth columns are results generated by our method.



**Figure 8:** Results generated by An et al. [3] and by our method. The first column give the original images; the second and third columns are results generated by An et al.; the fourth and fifth columns are results generated by our method.



**Figure 9:** Video results generated by Li et al. and our method. We have shown two frames of the video and clearly our method improves a lot along the boundaries.